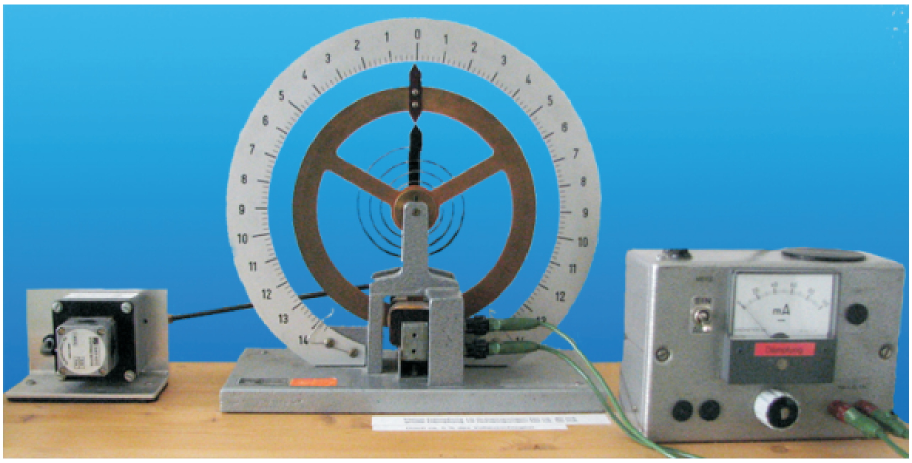
Experiment 13 - Resonance

PAP 1, [2] [1]

03.09.2024



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1. Introduction

1.1 Motivation

We investigate the dynamic properties of a rotary pendulum under the influence of damping and external excitation. Resonance phenomena are important in many physical systems, as they can lead to a sharp increase in amplitude if the excitation frequency corresponds to the ressonant frequency of the system. Understanding these effects is therefore fundamental to physics. The aim of the experiment is to analyze the behavior of a mechanical resonator under different conditions and to investigate the characteristics of free and forced oscillations as well as the effects of damping on the resonance curve.

1.2 Measurements

In this experiment, the oscillation behavior of a rotary pendulum is investigated and in particular the resonance phenomena. The pendulum is first allowed to swing freely in order to measure the undamped period of oscillation. It is then damped with an eddy current brake and the damping constant is determined. Finally, the pendulum is excited by an external force in order to investigate the dependence of the oscillation amplitude on the excitation frequency and to generate resonance curves. The influence of the damping on the resonance frequency, the maximum amplitude and the half-width of the resonance curve is analyzed.

1.3 Basics

1.3.1 Free Oscillation

In a free, damped oscillation, the amplitude of the oscillation decreases exponentially due to energy losses (e.g. through friction). The movement of the system can be described by the following equation:

$$a(t) = a_0 e^{-\delta t} \sin(\omega_f t) \tag{1.1}$$

Here is a_0 the initial amplitude, δ is the damping constant and $\omega_f$is the angular frequency of the damped oscillation.

1.3.2 Damping constant

The damping constant δ can be determined experimentally by plotting the amplitude of the oscillation logarithmically over time:

$$\delta = \frac{\ln(2)}{t_{1/2}} \tag{1.2}$$

where $t_{1/2}$is the time in which the amplitude has fallen to half its original value.

1.3.3 Forced oscillation

If an external periodic force is exerted on the system, this is referred to as forced oscillation. In this case, the amplitude of the oscillation depends strongly on the frequency of the external force and reaches its maximum when the excitation frequency corresponds to the natural frequency of the system. This is described by the resonance curve:

$$b(\omega) = \frac{A\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}} \tag{1.3}$$

Here is ω_0 the natural frequency of the undamped system, ω is the excitation frequency, and A is the amplitude of the excitation.

1.3.4 Resonant frequency

The resonant frequency ω' at which the amplitude is maximal can be determined by :

$$\omega' = \sqrt{\omega_0^2 - 2\delta^2} \tag{1.4}$$

In real systems, this frequency deviates only slightly from the undamped natural frequency.

1.3.5 Half-width

The half-width H of the resonance curve, which describes the width of the curve with not to strong damping at height $\frac{b(\omega')}{\sqrt{2}}$ is given by:

$$H = 2\delta \tag{1.5}$$

2. Means of Measurement

2.1 Measurements

Measurement protocol 3.9.2024

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Experiment 13 - Resonance

1. measurement setup:
   - rotary pendulum (Pohl's wheel), stimulated by a stepper motor with an eccentric
   - stepper motor drive with power supply
   - function generator
   - power supply unit for the eddy current brake

2. Sketch:

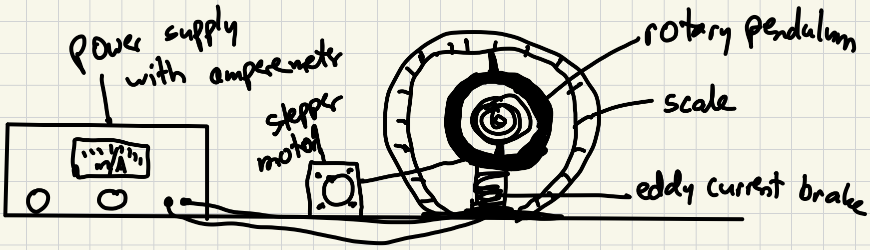

Table 1: Determination of the oscillation period T_0 of an undamped free rotary pendulum

Nr.	measurement (20 oscillations) [s]	T_0 (s)	$\bar{T}$ [s]	$\sigma_{\bar{T}}$ (s)
1	36,17	1,81		
2	36,23	1,81	1,81	0,00
3	36,27	1,81		

inaccuracy caused by reaction time: $\Delta T_0 = 0,2s$

$$\Delta\bar{T} = \sqrt{\left(\frac{\sigma_{\bar{T}}}{\sqrt{3}}\right)^2 + \left(\frac{\Delta T_0}{20}\right)^2}$$

$$\Delta\bar{T} = 0,01s$$

Table 2: Determination of the transient time for different damping

damping current [mA]	# oscillations	measured time [s]	oscillation period T_f [s]	ΔT_f [s]
340	15	26,08	1,74	0,013
440	10	17,92	1,79	0,02

reading error $\Delta A = 20mA$

we can't set the current precisely!

Table 3: deflection for 340 mA   reading error: Δdeflection = 0,2

Nr.	1	2	3	4	5	6	7	8	9
deflection	15,2	12,4	10,3	8,5	6,9	5,6	4,6	3,8	3,0
	15,1	12,5	10,3	8,4	6,9	5,7	4,5	3,8	2,9
mean	15,2	12,5	10,3	8,5	6,9	5,7	4,6	3,8	3,0
σ_{mean}	0,05	0,05	0	0,05	0	0,05	0,05	0	0,05

Nr.	10	11	12	13	14	15
deflection	2,4	1,9	1,4	1,0	0,7	0,4
	2,4	1,8	1,4	1,0	0,8	0,4
mean	2,4	1,9	1,4	1,0	0,8	0,4
σ_{mean}	0	0,05	0	0	0,05	0

Table 4: deflection for 440 mA

Nr.	1	2	3	4	5	6	7	8	9	10
deflection	13,4	9,4	6,8	4,8	3,4	2,3	1,5	0,9	0,5	0,2
	13,4	9,5	6,8	5,0	3,4	2,4	1,5	0,9	0,4	0,2
mean	13,4	9,5	6,8	4,9	3,4	2,4	1,5	0,9	0,5	0,2
σ_{mean}	0	0,05	0	0,1	0	0,05	0	0	0,05	0

Table 5: stationary amplitude for

für $I = 340$ mA $\quad$ $I = 440$ mA

total frequency	amplitude	amplitude
300	0,3	0,30
500	0,35	0,30
700	0,35	0,35
900	0,4	0,40
1100	0,45	0,50
1300	0,55	0,55
1500	0,70	0,70
1700	1,05	1,00
1850	2,00	1,70
1900	2,60	2,00
1950	3,50	2,30
2000	4,70	2,65
2050	4,20	2,50
2100	2,70	2,00
2150	2,10	1,70
2350	0,90	0,90
2550	0,60	0,55
2750	0,40	0,40

uncertainty of function generator frequency: $\Delta f = 5$ Hz

reading error: 0,2

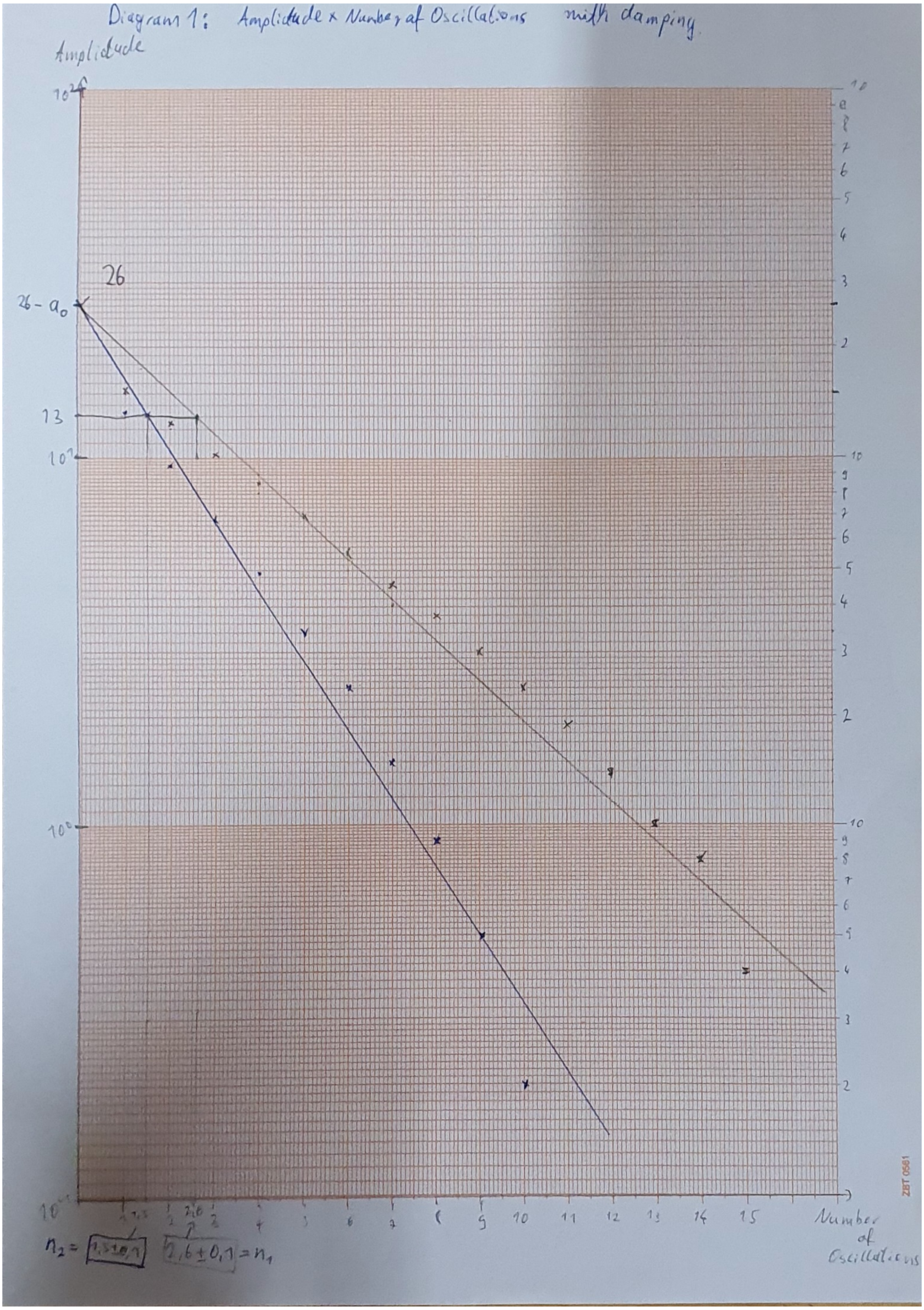

Diagram 1: Amplitude x Number of Oscillations with damping
Amplitude
26
26 - a_0
13
Number of Oscillations
n_2 = 1,5±0,1
2,6±0,1 = n_1

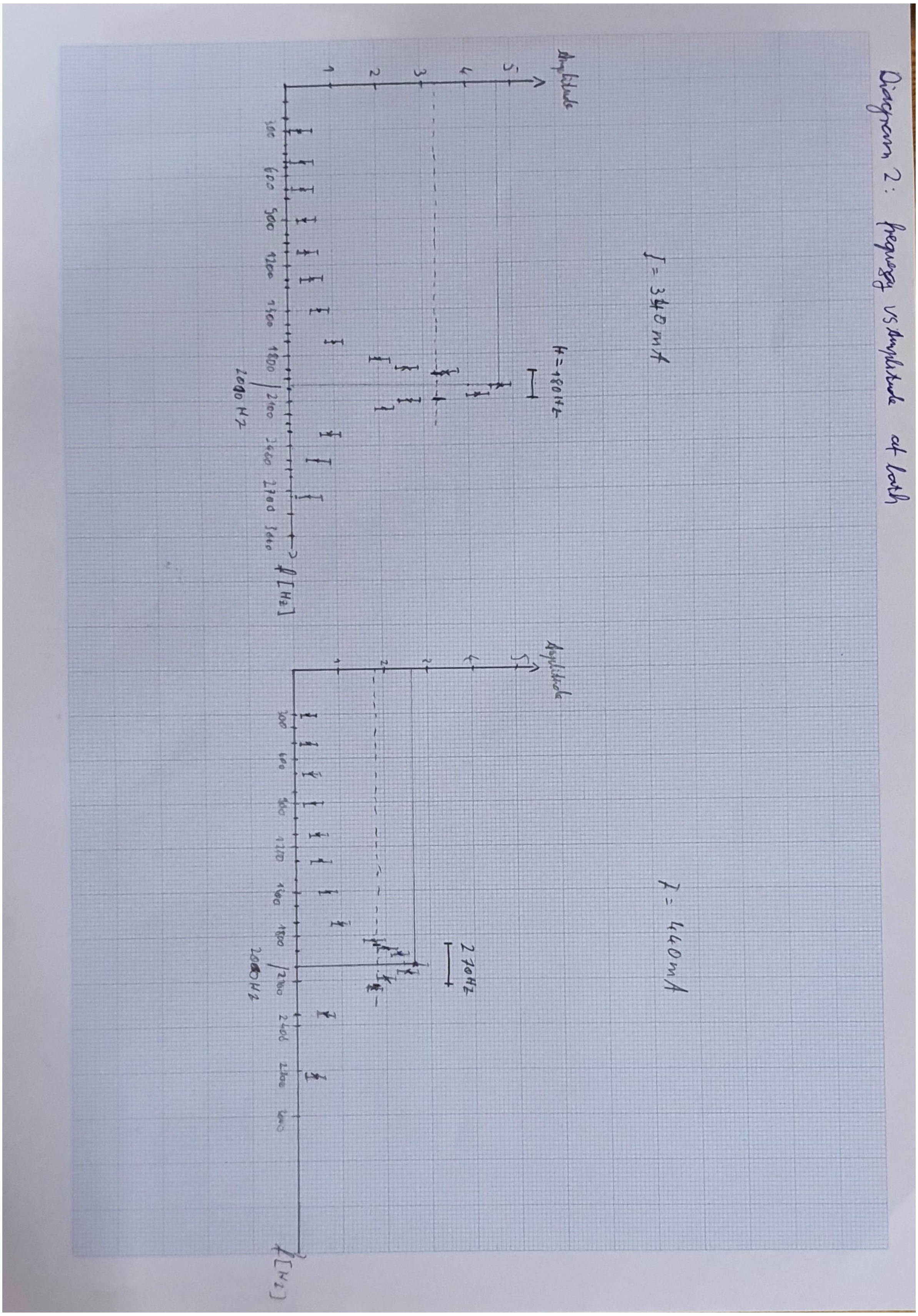
Diagram 2: frequency vs amplitude at both
I = 340 mA
I = 440 mA
Amplitude
f [Hz]
2000 Hz

3. Evaluation

3.1 Determination of T_0

We determine the period of oscilation by the following formula:

$$T_0 = \frac{\overline{T}}{n} \tag{3.1}$$

where $\overline{T}$ is the mean of the measured times, and n is the number of oscillations measured. In this case 20.

The uncertanty of the period time is calculated by:

$$\Delta T_0 = \sqrt{\Delta T^2_{\text{clock}} + \Delta T^2_{\text{stat}}} \tag{3.2}$$

Whereby ΔT_{clock} is the uncertanty of the stopped time, divided by 20. And ΔT_{stat} is the standart error of the mean.

With this we get a value of:

$$T_0 = (1,81 \pm 0,01)s$$

$$f_0 = (0,552 \pm 0,003)Hz$$

3.2 Determination of the damping constant

The Values of the Table 1 are plotted on a logarithmic scale, as a function of the number of oscillations.

From this we can get:

$$n_1 = (2,6 \pm 0,1) \text{ and } n_2 = (1,5 \pm 0,1)$$

Using Formula 1.2 and

$$t_{1/2} = T \cdot n_{1/2} \tag{3.3}$$

where $t_{1/2}$ is again the time in which the amplitude has fallen to half its original value, T is the oscillation period and $n_{1/2}$ is the Number of Oscillations it took, to half the Amplitude.

We obtain:

$$\delta_1 = (0,15 \pm 0,06)\frac{1}{s} \text{ for 340mA}$$

and

$$\delta_2 = (0,26 \pm 0,02)\frac{1}{s} \text{ for 440mA}$$

The error can be calculated with:

$$\Delta\delta = \sqrt{\left(\frac{-\ln(2)}{T^2 \cdot n_{1/2}} \cdot \Delta T\right)^2 + \left(\frac{-\ln(2)}{n_{1/2}^2 \cdot T} \cdot \Delta n_{1/2}\right)^2} \tag{3.4}$$

3.3 Period of Oscillation of dammed Penduluum

From Diagram 2 we can determine the frequency at which the Amplitude is highest, therefore it is the ressonante frequency.

In both cases this value is around 2010 ± 25 Hz, considering the factor $\frac{1}{4000}$ which is needed for the stepper motor frequency, this concludes to:

$$f_1 = (0,5025 \pm 0,006)Hz$$

3.4 Further Methods of evaluating the damping constant

With:

$$\omega_0 = \frac{2\pi}{T} \tag{3.5}$$

where ω_0 is the angular frequency of the undamped oscillaiton and T the period of Oscillation of the undamped and using formula 1.3.3 we can calculate the damping constant.

We get the following equation for δ:

$$\delta = \frac{\omega_0 \cdot b(\omega \to 0)}{2b(\omega')} \tag{3.6}$$

where ω' is the angular frequency of the measured peak, therefore $b(\omega')$ the amplitude value of the peak. For $b(\omega \to 0)$ we use our closest measument which was 0,3.

The error here can be calculated using:

$$\sqrt{\left(\frac{b(\omega \to 0)}{2b(\omega')} \cdot \Delta\omega_0\right)^2 + \left(\frac{\omega_0}{2b(\omega')} \cdot \Delta b(\omega \to 0)\right)^2 + \left(-\frac{\omega_0 b(\omega \to 0)}{2b(\omega')^2} \cdot \Delta b(\omega')\right)^2} \tag{3.7}$$

whereby $\Delta b(\omega')$ and $\Delta b(\omega \to 0)$ can be calculated using the added squares of the reading errors from the amplitude scale and the reading error from where the peak is and $\Delta\omega_0$ can be caltulated by:

$$\Delta\omega_0 = 2\pi \frac{\Delta T_0}{T_0^2} \tag{3.8}$$

Another method is using the Half-width H

This can be done using the formula 1.3.5 and reading the values for H from the Diagram 2 and then calculate the damping constant.

Damping	value from diagram for H	real H
340mA	(180 ± 25)Hz	$(0,283 \pm 0,039)$ Hz
440mA	(270 ± 25)Hz	$(0,424 \pm 0,039)$Hz

3.5 Values of δ

Damping in [mA]	δ_a in [Hz]	δ_b in [Hz]	δ_c in [Hz]
340	$0,15 \pm 0,06$	$(0,14 \pm 0,02)$	$(0,11 \pm 0,03)$
440	$0,26 \pm 0,02$	$(0,21 \pm 0,02)$	$(0,20 \pm 0,07)$

δ_a = first method, δ_b = Half-Width, δ_c = resonance amplification

3.6 Z-Values of Pairs of δ

Damping in [mA]	$\sigma_{a,b}$ in σ	$\sigma_{b,c}$ in σ	$\sigma_{a,c}$ in σ
340	0,158	0,831	0,596
440	1,768	0,137	0,824

4. Conclusion

In the first part of this experiment, we measured the period of an undamped rotary pendulum and were able to deduce a natural frequency of $f_0 = (0.552 \pm 0.003)$Hz.

We then damped the pendulum and determined the natural frequency again using a resonance curve. This was now $f_1 = (0.5025 \pm 0.006)$Hz. Here one can clearly see that the damping has increased the period duration and thus reduced the frequency.

It should be noted that we have obtained these two values using a different method and although a comparison is permissible, it is not as meaningful as a comparison of two numbers that were determined using the same method.

The Z-value of these two values is 7.3σ, which is why we can assume a significant difference, which one can present as well.

Furthermore, in this experiment we used several different methods to determine the damping constant of the rotary pendulum.

We were able to determine that all methods yielded values with a deviation of $< 2\sigma$. This means that every method is valid.

Bibliography

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